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RELAXATION FILTRATION OF HOMOGENEOUS FLUIDS IN CRACKED-POROUS MEDIA

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Equations of relaxation filtration of homogeneous fluids in cracked-porous media are derived. From a general system of equations, systems of "truncated" and simplified equations of filtration are obtained, for which the laws of smearing of pressure jumps are studied and the ranges of convergence of their solutions to the solution of the general system of equations are established.

Theoretical concepts of filtration of homogeneous fluids in cracked-porous media (CPMs) were given in [1, 2]. In conformity with this theory, cracked-porous media are represented as two coexistent interpenetrating continua (media) which have contrasting capacitive and filtration characteristics. The first medium represents a system of cracks, while the second one represents porous blocks. The equations of motion and mass conservation are written for each medium separately, i.e., two penetrabilities, two porosities, two filtration rates, and two pressures are introduced at each point. The flow of the fluid from one medium to the other is taken into account by introducing a source-sink function into the equations of mass conservation. It is assumed that the bed is homogeneous and isotropic and the flow in both media occurs within the limits of validity of Darcy's law. The fluid is weakly compressible; both media are elastic; we have fluid exchange between the cracks and the porous blocks, and the mass of the fluid flowing from the blocks into the cracks obeys the relation

$$q = \alpha_0 \frac{\rho_0}{\mu} (p_2 - p_1) , \tag{1}$$

where α_0 is the dimensionless coefficient that depends on the geometric characteristics of the porous blocks and ρ_0 is the density at the initial pressure p_0. On these premises the equations of filtration take the following form:

$$\chi \Delta p_1 = \varepsilon_1 \frac{\partial p_1}{\partial t} - \frac{p_2 - p_1}{\tau} , \quad \chi \varepsilon_2 \Delta p_2 = \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau} , \tag{2}$$

where

$$\chi = \frac{k_1}{\mu \beta_2^*} ; \quad \varepsilon_1 = \frac{\beta_1^*}{\beta_2^*} ; \quad \varepsilon_2 = \frac{k_2}{k_1} ; \quad \tau = \frac{\mu \beta_2^*}{\alpha_0} ;$$

$$\beta_i^* = \beta_{\mathrm{m}i} + m_{0i} \beta_{\mathrm{fl}} ; \quad \rho = \rho_0 \left[1 + \beta_{\mathrm{fl}} (p_i - p_0)\right] ; \quad m_i = m_{0i} + \beta_{\mathrm{m}i} (p_i - p_0) ; \quad \mathbf{v}_i = -\frac{k_i}{\mu} \nabla p_i , \quad i = 1, 2 .$$

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From the general model (2), under the conditions that the cracked porosity m_1 and the compressibility β_{m1} are low compared to these parameters of the porous blocks, whereas the penetrability of the porous blocks is low compared to that of the cracks, i.e., $m_1 << m_2$, $\beta_{m1} << \beta_{m2}$, and $k_2 << k_1$, a simplified system of equations is obtained:

$$\chi \Delta p_1 + \frac{p_2 - p_1}{\tau} = 0\,, \quad \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau} = 0\,. \tag{3}$$

In [3], J. E. Warren and P. J. Root took into account the compressibility of cracks but neglected the motion of the fluid in porous blocks. Under these assumptions, Eq. (2) yields the system of equations

$$\chi \Delta p_1 = \varepsilon_1 \frac{\partial p_1}{\partial t} - \frac{p_2 - p_1}{\tau}\,, \quad \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau} = 0\,, \tag{4}$$

which in certain publications is called "truncated."

The model described in [1, 2] is widely used in analyzing the processes of exploitation of oil deposits with cracked and cracked-porous collectors [4–6].

An analysis of the models of fluid motion in cracked-porous media can be found in [4, 7]. Formulation of the problems for the simplified and "truncated" systems of equations (3) and (4) has a number of special features information on which can be found in [7, 8]. Many researchers, in particular, Z. X. Chen [7], point to the fact that the simplified and "truncated" equations have some defects which must be taken into account in formulating the problems.

In the models presented, the fluid is considered to be viscous. However, as is well known, in many deposits with cracked-porous collectors oils possess non-Newtonian properties [9, 10] which during the motion of such fluids in porous and cracked-porous media can be manifested in the form of various anomalous effects. These can include, apart from the known nonlinearity of the filtration laws and manifestation of a limiting pressure gradient, a nonequilibrium coupling between the filtration rate and the pressure gradient. At the present time, an unambiguous coupling between the nonequilibrium condition of the filtration law and nonequilibrium rheological properties of the fluid has not been established. The investigations performed in [11–13] have shown that the macroscopic properties of cracked-porous media during the motion of viscous fluids in them depend on the characteristic dimensions of the pores and cracks and on the macroscopic medium itself. For certain relations of these scales the macroscopic medium manifests memory effects, which can be considered as a more general phenomenon than retardation effects in the model of [1, 2] caused by the mass exchange between the cracks and the porous blocks. This shows once more that relaxation phenomena in the filtration laws can be caused by a wider range of reasons than only by the nonequilibrium relaxation rheological properties of the fluid and the skeleton of the porous medium (or of the cracked-porous medium). The macroscopic law of filtration for an Oldroyd linear viscoelastic fluid in a porous medium is deduced in [14]. The results of this work indicate that the viscoelastic properties of the fluid can lead to various anomalous phenomena in the filtration law, in particular, to the enhancement of the relaxation properties of the filtration velocity and the pressure gradient. It is also shown that the nature of filtration is much more complicated than is given in [15–17] for the case of hypothetical models which establish the nonequilibrium coupling between the filtration rate and the pressure gradient.

Based on the phenomenological approach [1, 2], in the present work we derive the general equations of relaxation filtration in cracked-porous media, investigate the behavior of simplified and "truncated" relaxation systems of equations obtained from the general system, and estimate the ranges of convergence of their solutions to the solution of the general system.

We consider that the filtration laws in the cracks and the porous blocks have relaxation properties, with the characteristic relaxation times for them being different. Here the above-noted dependence of the filtration properties on the characteristic dimensions of the media is taken into account [18].

First, we consider the filtration law only with the pressure relaxation

$$\mathbf{v}_i = -\frac{k_i}{\mu}\left(1+\lambda_i \frac{\partial}{\partial t}\right)\text{grad}\, p_i\,, \quad i = 1, 2\,. \tag{5}$$

The equations of continuity in the phases are taken in the form [2]

$$-\,\text{div}\,(\rho \mathbf{v}_i) = \frac{\partial\,(m_i\,\rho)}{\partial t} \mp q\,, \quad i = 1, 2\,. \tag{6}$$

where q is the mass-exchange intensity between the phases; a minus sign before the quantity q corresponds to $i = 1$, while a plus sign corresponds to $i = 2$.

Using Eqs. (5) and (6), we obtain

$$\left(1+\lambda_i \frac{\partial}{\partial t}\right)\text{div}\left[\frac{\rho\,(p_i)\,k\,(p_i)}{\mu\,(p_i)}\,\text{grad}\, p_i\right] = \frac{\partial}{\partial t}\,[\rho\,(p_i)\,m\,(p_i)] \mp q\,, \quad i = 1, 2\,, \tag{7}$$

where q can be given in the form of Eq. (1).

Suppose that the fluid is compressible only weakly and volumetric relaxation effects are absent, i.e., $\rho = \rho_0[1 + \beta_{\text{fl}}(p_i - p_0)]$; both media are elastic, i.e., $m_i = m_{0i} + \beta_{\text{m}i}(p_i - p_0)$, $i = 1, 2$; the permeabilities are constant, $k_i = \text{const}$ and $\mu = \text{const}$ too. On these premises, from Eq. (7) we can come to the system

$$\frac{k_i}{\mu}\left(1+\lambda_i \frac{\partial}{\partial t}\right)\nabla^2 p_i = \beta_i^* \frac{\partial p_i}{\partial t} \mp \frac{\alpha_0}{\mu}\,(p_2 - p_1)\,, \tag{8}$$

where $\beta_i^* = \beta_{\text{m}i} + m_{0i}\beta_{\text{fl}}$, $i = 1, 2$.

Using the notation introduced in Eq. (2), we write system (8) in the form

$$\chi\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\nabla^2 p_1 = \varepsilon_1 \frac{\partial p_1}{\partial t} - \frac{p_2 - p_1}{\tau}\,, \quad \chi\varepsilon_2\left(1+\lambda_2 \frac{\partial}{\partial t}\right)\nabla^2 p_2 = \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau}\,. \tag{9}$$

On the assumption that $m_1 << m_2$ and $k_2 << k_1$, from Eq. (9) it is possible to obtain the following "truncated" system:

$$\chi\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\nabla^2 p_1 = \varepsilon_1 \frac{\partial p_1}{\partial t} - \frac{p_2 - p_1}{\tau}\,, \quad \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau} = 0\,, \tag{10}$$

which holds only for $\lambda_2 \frac{\partial}{\partial t} \sim E$, where E is the unit operator. For $\lambda_2 \frac{\partial}{\partial t} >> E$, but when $\varepsilon_2\lambda_2 \frac{\partial}{\partial t}$ is the significant operator, the second equation of system (10) has the form

$$\chi\varepsilon_2\lambda_2 \frac{\partial}{\partial t}\,\nabla^2 p_2 = \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau}\,. \tag{11}$$

Equations (10) will be called the Warren–Root relaxation system of equations.

As is evident from Eqs. (10) and (11), the influence of the relaxation effects (occurring in the porous blocks) in weakly nonstationary processes of flow is insignificant.

In a similar manner, we can derive equations in the case where the filtration law is prescribed in general form, i.e., in the form of hereditary integrals [9]

$$\mathbf{v}_i = -\frac{k_i}{\mu}\left(\text{grad } p_i + \int_0^t \Phi_i (t-\xi) \text{ grad } p_i d\xi\right)$$

or

$$\text{grad } p_i = -\frac{\mu}{k_i}\left(\mathbf{v}_i + \int_0^t F_i (t-\xi)\, \mathbf{v}_i \, d\xi\right), \quad i = 1, 2 .$$

The functions $\Phi_i(t)$ and $F_i(t)$ will be called the influence functions of the pressure (the pressure gradient) and the filtration rate, respectively. The filtration laws can also be written in the form of the integral rheological equations of the viscoelasticity theory [19] as

$$\mathbf{v}_i (t) = -\frac{k_i}{\mu}\int_{-\infty}^t \varphi_i (t-\xi) \frac{\partial}{\partial \xi} \nabla p_i (\mathbf{r}, \xi) \, d\xi \tag{12}$$

or

$$\nabla p_i (\mathbf{r}, t) = -\frac{\mu}{k_i} \int_{-\infty}^t f_i (t-\xi) \frac{\partial}{\partial \xi} \mathbf{v}_i (\xi) \, d\xi , \quad i = 1, 2 ,$$

and instead of Eq. (9) as

$$\begin{gathered} \chi \int_{-\infty}^t \varphi_1 (t-\xi) \frac{\partial}{\partial \xi} \nabla^2 p_1 (\mathbf{r}, \xi) \, d\xi = \varepsilon_1 \frac{\partial p_1}{\partial t} - \frac{p_2 - p_1}{\tau} , \\ \chi \varepsilon_2 \int_{-\infty}^t \varphi_2 (t-\xi) \frac{\partial}{\partial \xi} \nabla^2 p_2 (\mathbf{r}, \xi) \, d\xi = \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau} . \end{gathered} \tag{13}$$

From Eq. (13) for

$$\varphi_i = \left(1 + \frac{\lambda_i - \theta_i}{\theta_i} \exp\left(-\frac{t}{\theta_i}\right)\right) h (t) , \quad i = 1, 2 , \tag{14}$$

(where $h(t)$ is the Heaviside function) it is possible to derive the filtration equations that correspond to the law:

$$\mathbf{v}_i + \theta_i \frac{\partial \mathbf{v}_i}{\partial t} = -\frac{k_i}{\mu}\left(1 + \lambda_i \frac{\partial}{\partial t}\right) \text{grad } p_i , \quad i = 1, 2 .$$

In particular, substitution of Eq. (14) into Eq. (13) and passage to the limit for $\theta_i \to 0$, $i = 1, 2$, give Eq. (9).

From Eq. (13) we can obtain a "truncated" system of the type (10), whose working range is estimated by taking into account the significance of the integral operator

$$L\,(\bullet) = \int_{-\infty}^{t} \varphi_2\,(t-\xi)\,(\bullet)\,d\xi\,.$$

It should also be noted that on condition that $\beta_1^* << \beta_2^*$, the "truncated" system of equations (10) can yield the simplified system of equations

$$\chi \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \nabla^2 p_1 + \frac{p_2 - p_1}{\tau} = 0\,, \quad \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau} = 0\,. \tag{15}$$

Under the conditions that lead to Eq. (11), the second equation of the simplified system (15) is also replaced by Eq. (11).

As is shown in [20], the initial distributions of p_i, i = 1, 2, cannot be assigned arbitrarily. It is established that the discontinuities p_1 and $\partial p_1/\partial x$ are smeared instantly, i.e.,

$$[p_1] = 0\,, \quad \left[\frac{\partial p_1}{\partial x}\right] = 0\,, \tag{16}$$

whereas the discontinuities p_2 and $\partial p_2/dx$ are smeared exponentially, i.e.,

$$[p_2] = [p_2]_0 \exp\left(-\frac{t}{\tau}\right), \quad \left[\frac{\partial p_2}{\partial x}\right] = \left[\frac{\partial p_2}{\partial x}\right]_0 \exp\left(-\frac{t}{\tau}\right), \tag{17}$$

where $[f]$ is the discontinuity of the function f and $[p_2]_0$ and $\left[\dfrac{\partial p_2}{\partial x}\right]_0$ are the initial values of the discontinuities of p_2 and $\dfrac{\partial p_2}{\partial x}$.

Let us investigate the smearing behavior of the solution jumps of the relaxation equations of filtration in cracked-porous media (Eqs. (10) and (15)). To do this, we integrate the first equation of system (15) over the region $G_x\{-l \le x \le l\}$

$$\chi\tau \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \frac{\partial p_1}{\partial x}\Bigg|_{-l}^{l} + \int_{-l}^{l} (p_2 - p_1)\,dx = 0\,,$$

whence

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left[\frac{\partial p_1}{\partial x}\right] = 0\,. \tag{18}$$

Multiplying the same equation by x and then integrating it over the region G_x, we obtain

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) [p_1] = 0\,. \tag{19}$$

Equations (18) and (19) yield

$$[p_1] = [p_1]_0 \exp\left(-\frac{t}{\lambda_1}\right), \quad \left[\frac{\partial p_1}{\partial x}\right] = \left[\frac{\partial p_1}{\partial x}\right]_0 \exp\left(-\frac{t}{\lambda_1}\right). \tag{20}$$

where $[p_1]_0$ and $\left[\dfrac{\partial p_1}{\partial x}\right]_0$ are the initial values of p_1 and $\partial p_1/dx$, respectively.

Eliminating p_2 from Eq. (15), we find

$$\frac{\partial p_1}{\partial t} = \chi\left(1 + (\tau + \lambda_1)\frac{\partial}{\partial t} + \tau\lambda_1 \frac{\partial^2}{\partial t^2}\right)\Delta p_1 .$$

Eliminating p_1 from Eq. (15), we obtain quite an identical equation relative to p_2:

$$\frac{\partial p_2}{\partial t} = \chi\left(1 + (\tau + \lambda_1)\frac{\partial}{\partial t} + \tau\lambda_1 \frac{\partial^2}{\partial t^2}\right)\Delta p_2 . \tag{21}$$

To determine the smearing behavior of the jumps p_2 and $\partial p_2/\partial x$, we integrate Eq. (21) over the region G_x, which gives

$$\tau\lambda_1 \frac{\partial^2 P}{\partial t^2} + (\tau + \lambda_1)\frac{\partial P}{\partial t} + P = 0 , \quad P = \left[\frac{\partial p_2}{\partial x}\right]. \tag{22}$$

Multiplying Eq. (21) by x and integrating it again over the region G_x, we obtain the following equation, identical to Eq. (22), relative to $[p_2]$:

$$\tau\lambda_1 \frac{\partial^2 [p_2]}{\partial t^2} + (\tau + \lambda_1)\frac{\partial [p_2]}{\partial t} + [p_2] = 0 . \tag{23}$$

In order to investigate the behavior of solutions (23), we consider the following cases.

(1) Let $\lambda_1 \neq \tau$. Then

$$[p_2] = a \exp\left(-\frac{t}{\tau}\right) + b \exp\left(-\frac{t}{\lambda_1}\right), \tag{24}$$

where

$$a = -\tau(\lambda_1 - \tau)^{-1}(\lambda_1 [p_2]_0' + [p_2]_0) , \quad b = \lambda_1(\lambda_1 - \tau)^{-1}(\tau [p_2]_0' + [p_2]_0) , \quad [p_2]_0' = \left.\frac{\partial [p_2]}{\partial t}\right|_{t=0} .$$

(2) Let $\lambda_1 = \tau$. Then

$$[p_2] = \left([p_2]_0 + \left(\frac{[p_2]_0}{\tau} + [p_2]_0'\right)t\right)\exp\left(-\frac{t}{\tau}\right). \tag{25}$$

Solution (22) has the same form, as Eqs. (24) and (25), only instead of $[p_2]_0$ and $[p_2]_0'$ it is necessary to use $\left[\dfrac{\partial p_2}{\partial x}\right]_0$ and $\left[\dfrac{\partial p_2}{\partial x}\right]_0'$ $\left(\left[\dfrac{\partial p_2}{\partial x}\right]_0' = \left.\dfrac{\partial}{\partial t}\left[\dfrac{\partial p_2}{\partial x}\right]\right|_{t=0}\right)$, respectively.

Similarly, we can show that the smearing behavior of the solution jumps in the system of equations (10) is the same as in Eqs. (20), (24), and (25).

The solutions obtained indicate that in the case of relaxation filtration the jumps p_1 and $\partial p_1/\partial x$ are not attenuated instantly as this takes place in a classical model of filtration in cracked-porous media [20]. The smearing of the jumps occurs in conformity with the exponential law (20), where the characteristic smearing time is determined by the relaxation time of the pressure gradient, i.e., by λ_1. In the porous blocks, the smearing behavior of the solution jumps is also exponential. However, in contrast to the classical case, it is determined by the additive influence of two characteristic times of the process, i.e., by λ_1 and τ, while the scales of attenuations are determined by the values of λ_1 and τ. For $\lambda_1 >> \tau$ the more prolonged attenuation of the jumps is determined by the value of λ_1. Consequently, the attenuation of the jumps caused by the mass exchange between the porous blocks and cracks is completed relatively rapidly. The most distinctive behavior of smearing of the jumps is observed in the case $\lambda_1 = \tau$. Here the jumps p_2 and $\partial p_2/\partial x$ are smeared in conformity with the additive addition of purely exponential and power-exponential laws. In this case, for definite $t = t_* > 0$ the smearing of the jumps p_2 and $\partial p_2/\partial x$ can acquire a nonmonotonic character with one maximum (or minimum). Suppose that $[p_2]_0$ = const and $[p_2]_0'$ = const. Then

$$t_* = \frac{[p_2]_0\, \tau^2}{[p_2]_0 + \tau\, [p_2]_0'} .$$

When $\lambda_1 = 0$, from Eqs. (24) and (25) we obtain Eqs. (16) and (17), respectively.

Thus, the smearing of the solution jumps of the simplified and "truncated" systems of relaxation filtration equations in cracked-porous media is determined not only by the value of τ but also by λ_1. In certain situations, the nonmonotonic attenuation of the jumps of pressure and of its first derivative with respect to x in the porous blocks is possible. The attenuation of the jumps will have a unimodal character.

Now we estimate numerically the working ranges of the "truncated" and simplified systems of equations (10) and (15). For this, we consider the following model problem. Suppose that in a plane, semiinfinite, and homogeneous cracked-porous medium with initial pressure p_0 at the point $x = 0$ the pressure p_{m} = const is created. In accordance with this formulation, the initial and boundary conditions of the problem are written as follows:

$$p_i\,(0, x) = p_0\,, \quad 0 \le x < \infty\,; \quad p_i\,(t, 0) = p_{\mathrm{m}}\,; \quad p_i\,(t, \infty) = p_0\,, \quad i = 1, 2\,. \tag{26}$$

To solve this problem, we used the finite-difference method [21].

To approximate the equations in the region $D\{(x, t),\ 0 \le x < \infty,\ 0 \le t \le T\}$, we introduce a grid $\omega_{\theta h} = \{(x_k, t_j),\ k = 0, 1, 2, ...,\ j = 0, 1, 2, ..., J,\ x_k = kh,\ t_j = j\theta,\ \theta = T/J\}$, where T and J = const and h and θ are the grid steps for x and t, respectively. We introduce the notation $P1_k^j = p_1(t_j, x_k)$ and $P2_k^j = p_2(t_j, x_k)$.

System (9) is approximated by the implicit two-parameter finite-difference scheme

$$\begin{gathered}
\chi\left[\sigma_1 \Lambda P1_k^{j+1} + (1-\sigma_1)\,\Lambda P1_k^j + \frac{\lambda_1}{\theta}\,(\Lambda P1_k^{j+1} - \Lambda P1_k^j)\right] = \varepsilon_1\,\frac{P1_k^{j+1} - P1_k^j}{\theta} - \frac{P2_k^{j+1} - P1_k^{j+1}}{\tau}\,, \\
\chi\varepsilon_2\left[\sigma_2 \Lambda P2_k^{j+1} + (1-\sigma_2)\,\Lambda P2_k^j + \frac{\lambda_{12}}{\theta}\,(\Lambda P2_k^{j+1} - \Lambda P2_k^j)\right] = \frac{P2_k^{j+1} - P2_k^j}{\theta} + \frac{P2_k^{j+1} - P1_k^{j+1}}{\tau}\,,
\end{gathered} \tag{27}$$

where

$$\Lambda P1_k^j = \frac{1}{h^2}(P1_{k-1}^j - 2P1_k^j + P1_{k+1}^j)\ ; \quad \Lambda P2_k^j = \frac{1}{h^2}(P2_{k-1}^j - 2P2_k^j + P2_{k+1}^j)\ ;$$

σ_1 and σ_2 are the real parameters.

For definiteness, we consider purely implicit schemes: $\sigma_1 = \sigma_2 = 1$. Then Eq. (27) takes the form

$$\begin{aligned} &\chi\left[\Lambda P1_k^{j+1} + \frac{\lambda_1}{\theta}(\Lambda P1_k^{j+1} - \Lambda P1_k^j)\right] = \varepsilon_1 \frac{P1_k^{j+1} - P1_k^j}{\theta} - \frac{P2_k^{j+1} - P1_k^{j+1}}{\tau}\ , \\ &\chi\varepsilon_2\left[\Lambda P2_k^{j+1} + \frac{\lambda_2}{\theta}(\Lambda P2_k^{j+1} - \Lambda P2_k^j)\right] = \frac{P2_k^{j+1} - P2_k^j}{\theta} + \frac{P2_k^{j+1} - P1_k^{j+1}}{\tau}\ , \end{aligned} \tag{28}$$

System (28) gives the following difference equations:

$$a_k^1 P1_{k-1}^{j+1} + c_k^1 P1_k^{j+1} + b_k^1 P1_{k+1}^{j+1} + d_k^1 P2_k^{j+1} = -f_k^1\ , \quad a_k^2 P2_{k-1}^{j+1} + c_k^2 P2_k^{j+1} + b_k^2 P2_{k+1}^{j+1} + d_k^2 P1_k^{j+1} = -f_k^2\ , \tag{29}$$

where the coefficients a_k^1, a_k^2, b_k^1, b_k^2, c_k^1, c_k^2, f_k^1, and f_k^2 are expressed in terms of the known data of the problem.

The system of difference equations (29) is solved by the sweep method.

The "truncated" and simplified equations (10) and (15) are approximated as particular cases of general scheme (27).

Based on the numerical solution of the problem, we performed calculations for a certain set of initial values of the parameters m_i, k_i, and β_{mi}, $i = 1, 2$.

Figure 1 presents one variant of change in the absolute errors

$$\delta_{1n}(k, j) = \left|(P1_k^j)_1 - (P1_k^j)_n\right|\ , \quad \delta_{2n}(k, j) = \left|(P2_k^j)_1 - (P2_k^j)_n\right|\ ,$$

where $n = 2$, 3, 4, and 5.

The subscripts 1 and n correspond to the solutions of the systems of equations (9) (subscript 1), (10) ($n = 2$), and (15) ($n = 3$), and also to those of the corresponding systems of equations

$$\chi\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\nabla^2 p_1 = \varepsilon_1 \frac{\partial p_1}{\partial t} - \frac{p_2 - p_1}{\tau}\ , \quad \chi\varepsilon_2\lambda_2 \frac{\partial}{\partial t}\nabla^2 p_2 = \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau} \quad (n = 4) \tag{30}$$

and

$$\chi\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\nabla^2 p_1 + \frac{p_2 - p_1}{\tau} = 0\ , \quad \chi\varepsilon_2\lambda_2 \frac{\partial}{\partial t}\nabla^2 p_2 = \frac{\partial p_2}{\partial t} + \frac{p_2 - p_1}{\tau} \quad (n = 5)\ . \tag{31}$$

The analysis of all the variants of calculations has shown that in the case where β_{m1} and β_{m2} are comparable, the values of δ_{13} and δ_{23} are substantially larger than those of δ_{12} and δ_{22}. This indicates that the convergence of the solutions of the system of equations (15) does not satisfy the solutions of Eqs. (9). The small values of δ_{12} and δ_{22} show a good convergence between solutions of (9) and (10). At the points $x = 0$ and $x = L$, where L is the conditional boundary of the bed, we have $\delta_{in} = 0$ ($i = 1$, 2 and $n = 2$, 3), since the solutions of (9), (10), and (15) coincide in conformity with the boundary conditions.

In the case where $\beta_{m1} \ll \beta_{m2}$, the values of δ_{13} decrease noticeably compared to the previous case, which indicates a good convergence between the solutions of the systems of equations (9), (10), and (15).

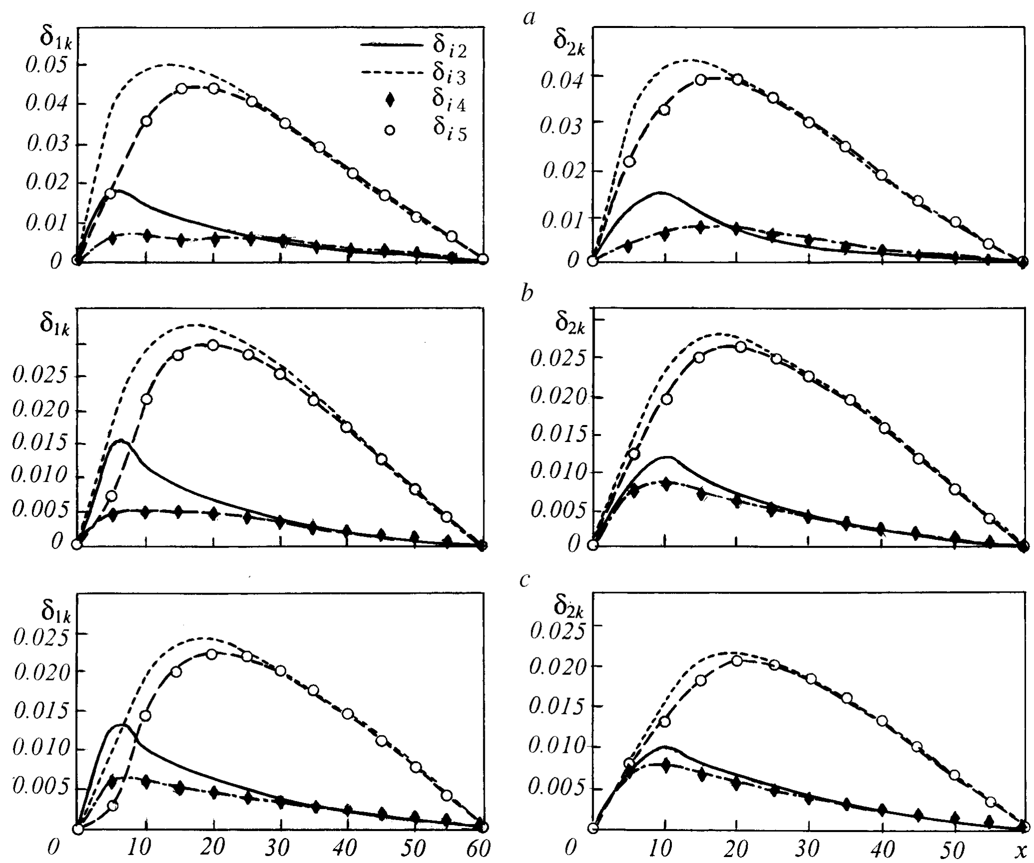

Fig. 1. Absolute errors δ_{in} (i = 1) for β_{m1} = $1.6 \cdot 10^{-5}$ and β_{m2} = $2.5 \cdot 10^{-5}$, MPa^{-1}, t = 100 sec, λ_1 = 40 sec (a), 100 (b), and 150 (c), λ_2 = 120 sec.

However, here, too, the values of δ_{13} and δ_{23} are much larger than the values of δ_{12} and δ_{22}. This means that the system of equations (15) gives solutions more distant from those of the system of equations (9).

Next, we consider the case where the influence of λ_2 is substantial. Just as above, when $\beta_{m1} \sim \beta_{m2}$, the quantities δ_{in} (i = 1, 2 and n = 3, 5) have larger values than δ_{in} (i = 1, 2 and n = 2, 4). This indicates a poor convergence of the solutions of the systems of equations (15) and (31) to the solution of (9), whereas the solutions of (10) and (30) have a comparatively better convergence. When $\beta_{m1} \ll \beta_{m2}$, the values of δ_{in} (i = 1, 2 and n = 3, 5) are substantially smaller than in the previous case, which means the improvement in the convergence of Eqs. (15) and (31) to the solution of the system of equations (9). When t are large, the solutions of (10) and (30) and (15) and (31) are close to each other. Considerable differences of δ_{i2} from δ_{i4} and δ_{i3} from δ_{i5} (i = 1, 2) are observed for small times t. Consequently, for small t, when the influence of λ_2 is substantial, it is necessary to use the solutions of the system of equations (30) and (31), whereas the use of the systems of equations (10) and (15) can lead to visible errors. Thus, it is established that in order to calculate p_1 and p_2 under the known assumptions, it is possible to use system (10) instead of the general system (9), but when $\beta_{m1} \ll \beta_{m2}$, it is worthwhile to resort to system (15). When the parameter λ exerts a considerable influence, systems (30) and (31) must be applied instead of systems (10) and (15).

The case of simultaneous action of the parameters λ_1 and λ_2 is also considered. Calculations were performed for the following values: λ_1 = 40, 100, and 150 sec and λ_2 = 120 sec. The calculation results show that in this case, too, the above-noted tendencies toward changing δ_{in} (i = 1, 2 and n = 2, 3, 4, 5) are retained.

Taking account of λ_1 and λ_2 leads to a substantial retardation of the decrease in the pressures in the systems of cracks and porous blocks that was noted earlier in [22].

Thus, using the simplified and "truncated" systems of equations of relaxation filtration in cracked-porous media, one must take into account the range of convergence of their solutions to the solution of the general system of equations.

It is easy to verify that for $\lambda_1 = \lambda_2 = 0$ systems (15) and (13) become a simplified system of Barenblatt and others [1, 2], while systems (10) and (30) become Warren–Root equations [3].

NOTATION

k_i, penetrability; m_i, porosity; m_{0i}, porosity for $p = p_0$; p_i, pressure; v_i, filtration rate; β_{fl}, coefficient of compressibility of the fluid; $\beta_{\text{m}i}$, coefficient of compressibility of the medium; μ, fluid viscosity; λ_1, relaxation time of the pressure gradient; θ_i, relaxation time of the filtration rate; ρ, fluid density; Δ, Laplace operator; τ, lag time caused by the mass exchange between the cracks and the porous blocks; ∇, Hamiltonian operator. Subscript $i = 1$ corresponds to the cracks; subscript $i = 2$ corresponds to the porous blocks.

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